Large and almost maximal neutrino mixing within the type II see-saw mechanism

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ABSTRACT: Within the type II see-saw mechanism the light neutrino mass matrix is given by a sum of a direct (or triplet) mass term and the conventional (type I) see-saw term. Both versions of the see-saw mechanism explain naturally small neutrino masses, but the type II scenario offers interesting additional possibilities to explain large or almost maximal or vanishing mixings which are discussed in this paper. We first introduce "type II enhancement" of neutrino mixing, where moderate cancellations between the two terms can lead to large neutrino mixing even if all individual mass matrices and terms generate small mixing. However, nearly maximal or vanishing mixings are not naturally explained in this way, unless there is a certain initial structure (symmetry) which enforces certain elements of the matrices to be identical or related in a special way. We therefore assume that the leading structure of the neutrino mass matrix is the triplet term and corresponds to zero U_{e3} and maximal θ_{23}. Small but necessary corrections are generated by the conventional see-saw term. Then we assume that one of the two terms corresponds to an extreme mixing scenario, such as bimaximal or tri-bimaximal mixing. Deviations from this scheme are introduced by the second term. One can mimic Quark-Lepton Complementarity in this way. Finally, we note that the neutrino mass matrix for tri-bimaximal mixing can be — depending on the mass hierarchy — written as a sum of two terms with simple structure. Their origin could be the two terms of type II see-saw.

KEYWORDS: Neutrino Physics, Beyond Standard Model.

Contents

1. Introduction

The smallness of neutrino masses arises naturally in the conventional or type I see-saw mechanism [1], with a low energy neutrino mass matrix of the form

$$m_\nu^I = -m_D^T \, M_R^{-1} \, m_D \; . \tag{1.1}$$

Here m_D is a Dirac mass matrix usually related to the known fermion masses or the weak scale $v \simeq 174\,\text{GeV}$, and M_R is a Majorana mass matrix of Standard Model singlet neutrinos with a mass scale M as large as the GUT scale. Hence, neutrino masses are naturally of order $v^2/M \sim 0.01\,\text{eV}$, corresponding nicely to the square root of the mass squared difference of atmospheric neutrinos. However, the other equally astonishing aspect of neutrino physics, namely the presence of large mixing angles, is a priori not explained by the type I see-saw mechanism. It is possible to generate large mixings by specific forms of the low energy mass matrix, and many models have been proposed [2] in order to explain the form of m_ν from the structure of m_D or of M_R, or of both of them.

In this article we want to discuss large mixings in the context of the type II see-saw mechanism [3], where the light neutrino mass matrix can be written as the conventional type I see-saw term plus an additional (triplet) contribution:

$$m_\nu = m_\nu^{II} + m_\nu^I = m_L - m_D^T \, M_R^{-1} \, m_D \; . \tag{1.2}$$

Since m_ν is now a sum of two terms, there are interesting non-trivial possibilities, not present in the conventional see-saw mechanism, which can naturally be related to large or nearly maximal mixings. The first suggestive option is, that for some reason, both terms could be of comparable magnitude and (moderate) cancellation is connected to the interesting features of neutrino mixing. Alternatively, it could be the sum of the two comparable terms which is crucial. Most naturally, one term dominates, while the other term introduces only a small correction. The interplay of both terms has so far been analyzed only in a few papers, for instance within specific SO(10) models [4], regarding the reconstruction of the mass matrices [5, 6], or in other scenarios [7 – 10]. Specifically, we focus our discussion in this paper in the context of the type II see-saw on four aspects of large neutrino mixing:

(i) we point out in section 3 that even if all involved matrices, and even both terms in eq. (1.2), generate small mixing, a moderate cancellation can generate large mixings in m_ν ("type II enhancement"). This happens if the involved matrices have similar or even identical flavor structure, which distinguishes the scenario from the usual (type I) see-saw enhancement of neutrino mixing. This mechanism produces typically sizable or large mixings, but maximal or exactly vanishing mixings are not expected (as in basically all models for lepton mixing) unless in addition certain elements of the matrices are related;

(ii) in order to explain naturally almost maximal or almost vanishing mixings, we propose in section 4 that one of the two terms in $m_L - m_D^T M_R^{-1} m_D$ is dominant and that it corresponds to $U_{e3} = 0$ and to maximal θ_{23}. The second term would then be responsible for small or tiny corrections;

(iii) in section 5 we assume that the triplet term in the type II see-saw formula corresponds to a specific mixing scheme, e.g., bimaximal or tri-bimaximal mixing. A subleading conventional term then introduces a perturbation to this mixing scheme, thereby explaining deviations from bimaximal or tri-bimaximal mixing. It is also possible to mimic Quark-Lepton Complementarity in this way;

(iv) in section 6 we finally take advantage of the fact that the neutrino mass matrix for tri-bimaximal mixing can almost always be written as a sum of two terms with simple structure. Their origin could be the two terms of type II see-saw.

To the best of our knowledge, the main points we make here have not been emphasized in the literature before. We will not construct explicit models for the issues given here, or conduct detailed numerical or analytical studies, but rather limit ourself to give instructive examples for each of these cases. We hope this will point the way to interesting model building possibilities and bring some attention to the various unexplored features of the type II see-saw. Before discussing the issues mentioned above, we will start by shortly summarizing the framework of the present study in the next section.

2. Framework

2.1 Neutrino mixing and the mass matrix

Let us shortly summarize the neutrino observables and our current knowledge about them. With the usual parametrization of the lepton mixing matrix (we neglect the phases in this work),

$$
U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} , \tag{2.1}
$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, we have as best-fit points [11] $U_{e3} = 0$, $\theta_{23} = \pi/4$ and $\sin^2\theta_{12} \simeq 0.3$. The zeroth order form of the mass matrix $m_\nu = U^* m_\nu^{\mathrm{diag}} U^\dagger$ can then be given as

$$
m_\nu = \sqrt{\frac{\Delta m_{\mathrm{A}}^2}{4}} \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} \text{ or } m_\nu = \sqrt{\frac{\Delta m_{\mathrm{A}}^2}{2}} \begin{pmatrix} 0 & 1 & 1 \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix} , \tag{2.2}
$$

when neutrinos obey a normal ($m_3^2 \gg m_{2,1}^2$) or inverted ($m_2^2 \simeq m_1^2 \gg m_3^2$ with m_1 and m_2 having opposite CP parities) hierarchy, respectively. Order one coefficients are not explicitly given here. The matrices in eq. (2.2) can for instance be obtained by asking for the conservation of the flavor charge L_e [12] or $L_e - L_\mu - L_\tau$ [13], respectively. Approximately, the dominating 23 block of the mass matrix can also be generated by sequential dominance of the right-handed neutrinos in type I see-saw scenarios [14]. If neutrinos are quasi-degenerate, $m_3 \simeq m_2 \simeq m_1 \equiv m_0$, there are also ways to explain this by simple symmetries. For instance, models based on SO(3) usually lead to a mass matrix proportional to the unit matrix [15], i.e., the three neutrinos all have the same CP parities. This is a rather unstable situation in what regards radiative corrections. Another possibility, along the lines of L_e and $L_e - L_\mu - L_\tau$, is

$$
m_\nu = m_0 \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 0 & 1 \\ \cdot & \cdot & 0 \end{pmatrix} , \tag{2.3}
$$

which corresponds to the conservation of $L_\mu - L_\tau$ [16, 17]. Apparently, some or all of the zero elements of these simple matrices in eqs. (2.2), (2.3) have to be filled with small entries. Alternatively, the flavor symmetries L_e, $L_e - L_\mu - L_\tau$ or $L_\mu - L_\tau$ have to be broken softly. One of the points we wish to make in this paper is that the type II see-saw mechanism with its two terms generating m_ν is a natural candidate to introduce the breaking parameters.

2.2 Origin of type II see-saw

The low energy neutrino mass matrix resulting from the type II see-saw is $m_\nu = m_\nu^{II} + m_\nu^{I} = m_L - m_D^T M_R^{-1} m_D$. The relevant Lagrangian is

$$
\mathcal{L} = \frac{1}{2} \overline{N_{Ri}} (M_R)_{ij} N_{Rj}^c + \frac{1}{2} \overline{L_\alpha^c} f_{\alpha\beta} \, i\tau_2 \, \Delta_L \, L_\beta + \frac{1}{v} \overline{N_{Ri}} (m_D)_{i\alpha} L_\alpha \Phi^\dagger , \tag{2.4}
$$

where N_{Ri} are the right-handed Majorana neutrinos and $L_\alpha = (\nu_\alpha, \alpha)_L^T$ is the lepton doublet with $\alpha = e, \mu, \tau$. There is also a Dirac mass matrix m_D governing the coupling of the Higgs doublet Φ with the N_{Ri}. The matrix m_ν^{II} stems from the second term in eq. (2.4) and requires a $SU(2)_L$ triplet, which can be written as

$$\Delta_L = \begin{pmatrix} \frac{1}{\sqrt{2}} \Delta^+ & \Delta^{++} \\ \Delta^0 & -\frac{1}{\sqrt{2}} \Delta^+ \end{pmatrix} . \qquad (2.5)$$

The neutral component develops a vacuum expectation value v_L, which together with the symmetric Yukawa coupling matrix $f_{\alpha\beta}$ gives a contribution $m_\nu^{II} = v_L f$ to the low energy neutrino mass matrix. The value of the ρ parameter and in particular the small neutrino masses imply that $v_L \ll v$. A popular scenario in which the type II see-saw can be realized is based on the left-right (LR) symmetric gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The LR gauge group is a subgroup of the Pati-Salam group and it can also be obtained from SO(10). Gauge symmetry implies the existence of $V - A$ and $V + A$ interactions. Moreover, gauge symmetry demands the presence of a $SU(2)_R$ Higgs triplet Δ_R. By developing a vacuum expectation value v_R, $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is broken down to the Standard Model. The mass matrix of the right-handed neutrinos $M_R = v_R g$ is also generated, where g is a symmetric Yukawa coupling matrix.[1] An even more appealing and interesting scenario occurs when in addition to the LR gauge symmetry there is a discrete LR symmetry, in which case m_L and M_R have identical flavor structure and are proportional to each other:

$$m_L \equiv v_L f = \frac{v_L}{v_R} M_R \quad \text{with } v_L v_R = \gamma v^2 , \qquad (2.6)$$

where γ is a model-dependent function of the underlying theory. The discrete LR symmetry of the form $f = g$ implies in addition that m_D is symmetric. The Yukawa matrix f defines the flavor structures of both m_L and M_R. Using $v_L v_R = \gamma v^2$ we have

$$m_\nu = v_L \left(f - m_D^T \frac{f^{-1}}{\gamma v^2} m_D \right) . \qquad (2.7)$$

It is apparent that the relative magnitude of the two terms in m_ν depends on γ. In order to have both terms in the type II see-saw formula to be of similar magnitude, and with assuming that at least one entry of m_D is of order v, the value $\gamma = \mathcal{O}(1)$ suggests itself.[2] Moreover, if one entry of m_D is of order v, dominance of the conventional see-saw term corresponds to $\gamma \ll 1$, whereas dominance of the triplet term corresponds to $\gamma \gg 1$. In the limit of $v_R \to \infty$ the parameter v_L and therefore the neutrino mass goes to zero. In addition, the theory becomes purely $V - A$. Hence, such theories relate the smallness of neutrino masses with the maximal parity violation of the weak interactions, a feature which makes them from an esthetical point of view very attractive. Actually, to have this connection a LR gauge symmetry suffices and no need for a discrete symmetry is present. In

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[1] The Higgs doublet now becomes a bi-doublet, which however does not affect our discussion.

[2] Actually, the situation is slightly more complicated [6], but this simplified discussion suffices to emphasize the main points.

fact, for most of the issues to be discussed in the following, neither a LR gauge nor discrete symmetry are necessary. From the model building point of view it is however interesting to see where one could afford such a symmetric and esthetical framework. Moreover, gauge and discrete LR symmetry reduce the number of free parameters and simplify the analysis. Some cases to be presented will however not be possible when a discrete left-right symmetry is present.

The possibility that a term is added to the conventional see-saw term $m_D^T M_R^{-1} m_D$ is of course not exclusively reserved for a Higgs triplet. There can be $B - L$ breaking dimension five operators from various sources [18], including Planck scale effects, SUSY contributions, radiative models, etc. All of these possibilities have their theoretical justification, and in principle our considerations can apply to these contributions, too.

3. From small to large mixing via type II see-saw

In this section we remark that in the type II see-saw mechanism moderate cancellation can lead to the generation of large neutrino mixing. This mechanism, which we call "type II enhancement" of neutrino mixing, can work successfully even if m_D, m_L, M_R and $m_D^T M_R^{-1} m_D$ correspond to small mixing. In fact, we will assume here that all individual matrices possess a "hierarchical masses with small mixing" form. This has its motivation in the large hierarchy of the charged lepton and quark masses, as well as the small quark mixing.

Let us first recall the generation of large mixing from small mixing in case of the conventional see-saw mechanism.

3.1 The situation in the type I see-saw

Before experimental results made a paradigm change necessary, one expected that there is some form of quark-lepton symmetry which then implies that lepton mixing — in analogy to quark mixing – is described by small mixing angles. However, after the discovery of large lepton mixing it turned out that in principle one can generate large mixing in m_ν^I from small mixing in m_D and M_R by appropriate choice of the hierarchies in, and parameters of, the matrices m_D and M_R [19]. For instance, in a simple 2-flavor framework the mass matrices could be

$$m_D = v \begin{pmatrix} \epsilon_D & a\,\epsilon_D \\ b\,\epsilon_D & 1 \end{pmatrix} \text{ and } M_R = M \begin{pmatrix} \epsilon_M & 0 \\ \cdot & 1 \end{pmatrix}, \qquad (3.1)$$

with $\epsilon_{D,M} \ll 1$ and $a, b = \mathcal{O}(1)$. The individual mixing angles of these two matrices are small or even zero. The relevant parameter for the relative hierarchy between m_D and M_R is $\eta \equiv \epsilon_D^2/\epsilon_M$. In case of $\eta \gg 1$, or $\epsilon_D^2 \gg \epsilon_M$, the mixing angle for $m_\nu = -m_D^T M_R^{-1} m_D$ is large:

$$m_\nu = -\frac{v^2}{M} \begin{pmatrix} \eta + b^2\,\epsilon_D^2 & a\,\eta + b\,\epsilon_D \\ \cdot & 1 + a^2\,\eta \end{pmatrix} \xRightarrow{\eta \gg 1} \tan 2\theta \simeq \frac{2}{a - 1/a} = \mathcal{O}(1). \qquad (3.2)$$

Note that the individual mixings of m_D and M_R are small – in analogy to the quark sector — but the mixing of m_ν is large. This "see-saw enhancement" can be traced to $\epsilon_D^2 \gg \epsilon_M$, i.e., a stronger hierarchy in the Majorana sector [19]. Naively, one might say that the hierarchy of m_D is squared in the type I see-saw formula, so that the hierarchy in M_R has to be strong to cancel it. Note that we have assumed (close to) symmetric m_D, as implied for instance by a discrete LR symmetry. If the symmetry basis is not the basis in which the charged lepton mass matrix m_ℓ is real and diagonal, then m_D will be slightly non-symmetric if the matrix diagonalizing m_ℓ contains only small mixing angles. Our arguments would remain valid in this case. We should remark here that for highly non-symmetric Dirac mass matrices it is possible to generate successful large neutrino mixing even if M_R and m_D have very similar hierarchy [20]. To generate maximal mixing from eq. (3.2) one would require $a = 1$, which means that two entries in m_D are identical. The equality of certain elements is always necessary for extreme mixing angles. We can generalize the procedure to three generations. Suppose that m_D is "up-quark-like", i.e., it contains masses (in units of v) of order 1, ϵ_D^2 and ϵ_D^4:

$$
m_D = v \begin{pmatrix} \epsilon_D^4 & a\,\epsilon_D^3 & b\,\epsilon_D^3 \\ a\,\epsilon_D^3 & c\,\epsilon_D^2 & d\,\epsilon_D^2 \\ b\,\epsilon_D^3 & d\,\epsilon_D^2 & 1 \end{pmatrix} \text{ and } M_R = M \begin{pmatrix} \epsilon_{M1} & 0 & 0 \\ \cdot & \epsilon_{M2} & 0 \\ \cdot & \cdot & 1 \end{pmatrix}, \tag{3.3}
$$

where the diagonal M_R is described by two small parameters ϵ_{M1} and ϵ_{M2}. The light neutrino mass matrix is

$$
m_\nu = -\frac{v^2}{M} \begin{pmatrix} \epsilon_D^2 \left(b^2 \epsilon_D^4 + \eta_1 + a^2 \eta_2\right) & \epsilon_D \left(b\,d \epsilon_D^4 + a\,\eta_1 + a\,c\,\eta_2\right) & \epsilon_D \left(b\,\epsilon_D^2 + b\,\eta_1 + a\,d\,\eta_2\right) \\ \cdot & d^2 \epsilon_D^4 + a^2 \eta_1 + c^2 \eta_2 & d\,\epsilon_D^2 + a\,b\,\eta_1 + d\,c\,\eta_2 \\ \cdot & \cdot & 1 + b^2 \eta_1 + d^2 \eta_2 \end{pmatrix}, \tag{3.4}
$$

where we defined $\eta_1 = \epsilon_D^6/\epsilon_{M1}$ and $\eta_2 = \epsilon_D^4/\epsilon_{M2}$. To have a dominating 23 block in this matrix, we can either have $\eta_2 \gg \eta_1, \epsilon_D^2$ or $\eta_1 \gg \eta_2, \epsilon_D^2$. In the first case we have

$$
m_\nu = -\frac{v^2}{M} \begin{pmatrix} a^2 \epsilon_D^2 \eta_2 & a\,c\,\epsilon_D\,\eta_2 & a\,d\,\epsilon_D\,\eta_2 \\ \cdot & c^2 \eta_2 & d\,c\,\eta_2 \\ \cdot & \cdot & 1 + d^2 \eta_2 \end{pmatrix}, \tag{3.5}
$$

which for η_2 of order (or larger than one) is the wanted leading order structure of m_ν. Note that maximal 23 mixing in case of η_2 larger than one would require $d = c$, i.e., equality of certain mass matrix elements. Realistic predictions require corrections to this matrix from the remaining Majorana masses via η_1 [14]. We stress again that this see-saw enhancement of the mixing requires that the hierarchy in M_R is stronger, or the mixing is smaller, than that in the (close to symmetric) m_D. Consequently, if M_R and m_D have a similar flavor structure, and hence similar small mixing angles, then such a procedure is doomed. As we will argue in the following, in the type II see-saw case there is no problem in this case.

3.2 The situation in the type II see-saw

We will show now that the peculiar interplay of the two terms in the type II see-saw formula can give large mixing even if M_R and m_D have small mixing of the same order of magnitude ("type II enhancement"). The need to construct models in which the flavor structure of the right-handed neutrinos is very much different from the one of the other fermions is therefore absent. What we essentially note is that if both m_ν^{II} and m_ν^I generate small mixing (as in the quark sector), their sum does not necessarily need to do so and can correspond to large neutrino mixing. The conditions under which this can occur are outlined below, but the essential requirement in our example is only a moderate cancellation in the 33 entry of m_ν.

Let us demonstrate the idea in a simple 2-neutrino framework: consider a hierarchical Dirac mass matrix of the form

$$m_D = v \begin{pmatrix} a_D \, \lambda^4 & b_D \, \lambda \\ b_D \, \lambda & 1 \end{pmatrix} . \tag{3.6}$$

For simplicity, we have chosen here m_D to be symmetric, an assumption which by no means affects the validity of our argument. Since the mechanism is working for similar flavor structures of the involved matrices, we do not introduce small ϵ_D for m_D and ϵ_M for M_R, but rather parametrize the matrices in terms of a single small parameter λ, which can be thought of to be of the order of the Cabibbo angle. We choose a Majorana mass term for the right-handed neutrinos with similar hierarchy:[3]

$$M_R = v_R \begin{pmatrix} a_R \, \lambda^3 & b_R \, \lambda \\ \cdot & 1 \end{pmatrix} . \tag{3.7}$$

We introduced real parameters a_c and b_c (with $c = D, R$ for the Dirac and Majorana mass matrix, respectively), which are of order one. Within the conventional see-saw mechanism, we have (giving only the lowest powers of λ)

$$m_\nu^I = -m_D^T \, M_R^{-1} \, m_D \simeq \frac{v^2}{v_R} \frac{b_D(b_D - 2b_R)}{b_R^2} \begin{pmatrix} \frac{a_R \, b_D}{b_D - 2b_R} \lambda^3 & \frac{-b_D \, b_R}{b_D - 2b_R} \lambda \\ \cdot & 1 \end{pmatrix} , \tag{3.8}$$

which generates small mixing, $\theta^I = \mathcal{O}(\lambda)$. The mixing angles for m_D and M_R, respectively, are also of the same order: $\theta^D \simeq \theta^R = \mathcal{O}(\lambda)$. Let us assume a discrete LR symmetry. Then, with $m^{II} \propto M_R$ we also have that the mixing of the triplet term is small: $\theta^{II} = \mathcal{O}(\lambda)$. Now we add m_ν^I to m_ν^{II}, which yields

$$m_\nu = m_\nu^{II} + m_\nu^I \simeq v_L \begin{pmatrix} \left(a_R + \frac{a_R \, b_D^2}{b_R^2 \, \gamma} \right) \lambda^3 & \left(b_R - \frac{b_D^2}{b_R \, \gamma} \right) \lambda \\ \cdot & 1 + \frac{b_D \, (b_D - 2b_R)}{b_R^2 \, \gamma} + \frac{a_R \, (b_D - b_R)^2}{b_R^4 \, \gamma} \lambda \end{pmatrix} . \tag{3.9}$$

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[3]If m_D and M_R had identical flavor structure, our argument would still work but the resulting formulae would become longer. We comment below on an interesting aspect of these scenarios.

In general, this matrix generates two eigenvalues of order 1 and λ^2 and a small mixing angle of order λ. The crucial observation for type II enhancement is the following: suppose the term of order 1 in the 22 element of m_ν in eq. (3.9) cancels. In this case one has large mixing given by

$$\tan 2\theta \simeq \frac{4\, b_D\, b_R^3}{a_R\, (b_R - b_D)} = \mathcal{O}(1)\ . \tag{3.10}$$

If in addition $b_D = b_R$ holds we can even generate maximal mixing. Note that this formula holds when the order one term in eq. (3.9) cancels exactly. In order to generate large mixing (i.e., $\tan 2\theta$ of order 1) it suffices however that the cancellation of the two terms generates a term of order λ. If the 11 entry is very small, it is crucial that cancellations make the 22 entry of $m_\nu^{II} + m_\nu^{I}$ have the same order as the 12 entry. In order not to ask for too strong cancellation (and therefore fine-tuning) there should before cancellation be only one order of magnitude difference between the 12 and 22 entry. We have assumed discrete LR symmetry, forcing m_L and M_R to be proportional to each other. This is obviously not necessary to make the mechanism work.

In the realistic case of three generations we wish to obtain now the leading structure of the low energy mass matrix corresponding to a normal hierarchy. It is therefore necessary that, after the cancellation, the lower 23 block of m_ν has elements of the same order of magnitude, but larger than the entries in the first row. One may choose the following structures of the mass matrices:

$$m_D = v \begin{pmatrix} a_D\, \lambda^4 & b_D\, \lambda^3 & c_D\, \lambda^3 \\ \cdot & d_D\, \lambda^2 & e_D\, \lambda^2 \\ \cdot & \cdot & f_D \end{pmatrix} \text{ and } M_R = v_R \begin{pmatrix} a_R\, \lambda^3 & b_R\, \lambda^2 & c_R\, \lambda^2 \\ \cdot & d_R\, \lambda & e_R\, \lambda \\ \cdot & \cdot & f_R \end{pmatrix}\ . \tag{3.11}$$

We also choose discrete LR symmetry which means here $m_L = v_L\, M_R / v_R$. The mass spectrum of m_D is "up-quark-like", i.e., it contains masses (in units of v) of order 1, λ^2 and λ^4, while the eigenvalues of M_R (m_L) are in units of v_R (v_L) of order 1, λ and λ^3. The two mass spectra of m_D and M_R are therefore similar, and small mixing is predicted by both matrices. The structure of the mass matrix in the conventional see-saw mechanism is

$$m_\nu^{I} \simeq \frac{v^2}{v_R} \begin{pmatrix} \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^3) \\ \cdot & \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) \\ \cdot & \cdot & \frac{f_D^2}{f_R} + \mathcal{O}(\lambda) \end{pmatrix}\ , \tag{3.12}$$

which can not reproduce the neutrino data. Hence, if only the conventional type I see-saw term or only the triplet term m_L would contribute, then small neutrino mixing not capable

of explaining the data would result. However, the total neutrino mass matrix reads

$$m_\nu = m_\nu^{II} + m_\nu^I \simeq v_L \begin{pmatrix} a_R\,\lambda^3 & b_R\,\lambda^2 & c_R\,\lambda^2 \\ \cdot & d_R\,\lambda & e_R\,\lambda \\ \cdot & \cdot & (f_R - \frac{f_D^2}{f_R\gamma}) + \tilde{f}\,\lambda \end{pmatrix},$$

(3.13)

with $\tilde{f} = \dfrac{\left(c_R^2\,d_R - 2\,b_R\,c_R\,e_R + a_R\,e_R^2\right)\,f_D^2}{\left(b_R^2 - a_R\,d_R\right)\,f_R^2\,\gamma}$.

Let us again assume that the order one term in the 33 entry of eq. (3.13) cancels completely (again, it suffices that cancellation occurs just down to order λ). The condition for exact cancellation is quite simple, namely $f_R^2 = f_D^2/\gamma$, which is in fact simpler than the corresponding condition from the 2-flavor case discussed above. Then the mass matrix takes a well-known texture

$$m_\nu \simeq v_L\,\lambda \begin{pmatrix} a_R\,\lambda^2 & b_R\,\lambda & c_R\,\lambda \\ \cdot & d_R & e_R \\ \cdot & \cdot & \tilde{f} \end{pmatrix},$$

(3.14)

where the leading 23 block with entries of equal magnitude is necessary for large atmospheric mixing. The phenomenological consequences of eq. (3.14) are a normal mass hierarchy with $m_3^2 \gg m_{2,1}^2$. Thereby renormalization effects are rendered subleading [21], unless in the MSSM with very large $\tan\beta$ (see below). Moreover,

$$|U_{e3}| \sim \lambda \sim \sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}},$$

(3.15)

where $\Delta m_\odot^2 \simeq 8 \cdot 10^{-5}\,\mathrm{eV}^2$ ($\Delta m_A^2 \simeq 2 \cdot 10^{-3}\,\mathrm{eV}^2$) governs the oscillations of solar and long base-line reactor (atmospheric and long-baseline accelerator) neutrinos. This fixes the magnitude of $\lambda \simeq 0.2$ and of $v_L \simeq \sqrt{\Delta m_A^2}/\lambda \simeq 0.2\,\mathrm{eV}$. Neutrinoless double beta decay is suppressed and triggered by a small effective mass of order $|m_{ee}| \sim \sqrt{\Delta m_A^2}\,|U_{e3}| \sim \sqrt{\Delta m_\odot^2}$. The sizable $|U_{e3}|$ of order λ, a value close to current limits, is easily measurable in upcoming long-baseline or reactor oscillation experiments [22]. Moreover, atmospheric neutrino mixing deviates sizably from maximal, $\tan 2\theta_{23} \simeq 2\,e_R/(\tilde{f} - d_R)$. This will be testable with future precision data, too. Without additional symmetries forcing some elements of m_ν to be equal, neither zero $|U_{e3}|$ nor maximal θ_{23} can be achieved in this framework. To be precise, in eq. (3.14) one would need $b_R = c_R$ and $d_R = \tilde{f}$. Other aspects of type II see-saw, to be discussed in the next sections, could be used to achieve extreme values of mixing angles.

As is well known [12], the sub-determinant of the lower right 23 block has to be of order λ to generate a large solar neutrino mixing angle θ_{12}. Therefore, two mild cancellations to order λ are required: (i) the leading term in the 33 entry of m_ν has to cancel to order λ; (ii) the lower right 23 sub-determinant of m_ν has to be of order λ to generate large solar

neutrino mixing. The fact that two cancellations are required to make θ_{12} large, but only one to make θ_{23} large, could be used as an explanation why atmospheric neutrino mixing is larger than solar.

One may wonder whether one can generate the inverted hierarchy along similar lines. Here the requirement is that the 12 and 13 entries of m_ν are much larger than the other ones. When all individual matrices m_L, M_R and m_D correspond to small mixing, this would be rather unnatural since it requires cancellation within several independent elements of the resulting m_ν. Similar statements can be made for quasi-degenerate neutrinos. Note that we have chosen m_L in a way that before cancellation there is only one order of magnitude difference between the 33 and the 22,23 elements of m_ν. This guarantees that cancellation is necessary only for one entry. A more extreme example for cancellation in several elements can be found in ref. [8]: a discrete LR symmetric type II see-saw model based on $S(3)_L \times S(3)_R$ was considered, which allows two terms for each Majorana mass matrix, one term proportional to the unit matrix and one proportional to the democratic matrix. The latter term appears in m_ν^I and m_ν^{II} and has to cancel in order to generate large neutrino mixing.

The condition for the 33 entry in m_ν of eq. (3.13) to cancel down to order λ can be written as $f_R - \frac{f_D^2}{f_R \gamma} \overset{!}{=} a\,\lambda$. One may wonder whether radiative corrections can lead to this condition. For the normal hierarchy and within the Standard Model the radiative effects are always negligible below the see-saw scale. This can change in case of the MSSM, however. The effect of radiative corrections below the see-saw scale is to multiply the 13 and 23 element with $(1 + \epsilon)$ and the 33 element with $(1 + \epsilon)^2$, where

$$\epsilon \simeq -(1 + \tan^2 \beta) \frac{m_\tau^2}{16\,\pi^2\,v^2} \ln \frac{M_X}{M_Z} \simeq -2 \cdot 10^{-5} \,(1 + \tan^2 \beta)\,,$$

with m_τ the tau lepton mass. Large values of $\tan\beta \gtrsim 50$ could lead to a sizable correction $(1+\epsilon) \sim \lambda$, but would cause this on all elements of the third column of m_ν, in particular on the 23 entry. Hence, we cannot blame radiative effects for the generation of large mixing in the framework under study.

Our examples had slightly different textures for m_D and $M_R \propto m_L$. However, one could imagine cases in which all matrices have identical powers of λ in all entries. This implies an interesting aspect for the complete 6×6 neutrino mass matrix whose diagonalization will lead to eq. (1.2):

$$\mathcal{M}_\nu = \begin{pmatrix} m_L & m_D \\ m_D^T & M_R \end{pmatrix}\,. \tag{3.16}$$

It is easy to show that if the structures of $m_L \propto M_R$ and m_D are identical, i.e. $a_D = a_R$, $b_D = b_R$ and so on, then the determinant of $\mathcal{M}_\nu$ vanishes for $\gamma = 1$. The requirement for this is therefore that all matrices m_L, M_R and m_D are identical and only differ by their scales v_L, v_R and v. In addition, the exact relation $v_L\,v_R = v^2$ must hold.

4. Leading structures for zero U_{e3} and maximal θ_{23}

As mentioned above, with the type II see-saw enhancement discussed in the last section, it

is in general not possible to generate exactly maximal or zero mixing. Therefore, we will now assume that the leading structure of the neutrino mass matrix corresponds to zero U_{e3} and maximal θ_{23} and is provided by one of the terms in the type II see-saw formula. Small corrections are supplied by the other term, which can be either subleading or of similar magnitude (a study with no corrections from the conventional see-saw term is given in [24]). Let us recapitulate (see also [23] for the first two examples) the three simple, stable and often used candidates for zero U_{e3} and maximal θ_{23}:

$$\text{(A)}: \sqrt{\frac{\Delta m_{\text{A}}^2}{4}} \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{pmatrix}, \ \text{(B)}: \sqrt{\frac{\Delta m_{\text{A}}^2}{2}} \begin{pmatrix} 0 & 1 & 1 \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix}, \ \text{(C)}: m_0 \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 0 & 1 \\ \cdot & \cdot & 0 \end{pmatrix}. \tag{4.1}$$

They conserve the flavor charges L_e, $L_e - L_\mu - L_\tau$ and $L_\mu - L_\tau$, respectively. All three matrices have one eigenvalue with an eigenvector $(0, -1/\sqrt{2}, 1/\sqrt{2})^T$. This eigenvalue is $\sqrt{\Delta m_{\text{A}}^2}$ for case (A), 0 for case (B) and $-m_0$ for case (C). Therefore, they correspond to the normal hierarchy, the inverted hierarchy and quasi-degenerate neutrinos, respectively. Applying corrections to the three candidates is essential, since in their present form (A) and (B) have no solar Δm^2 while case (C) has no atmospheric Δm^2. Case (B) predicts maximal θ_{12}, the other candidates have no physical 12 mixing. The matrices in eq. (4.1) are exact, i.e., there are no order one coefficients involved. This is essential to have an eigenvalue of the form $(0, -1/\sqrt{2}, 1/\sqrt{2})^T$ except for matrix (C). This is because $L_\mu - L_\tau$ is the only allowed U(1) which is automatically μ–τ symmetric [25].

One appealing possibility is that these simple matrices correspond to the triplet term m_L and a small perturbation stems from the conventional see-saw term.[4] We thus assume that some symmetry enforces the triplet term to have one of the simple forms given in eq. (4.1). The leading structures (A), (B) and (C) could also stem from the conventional see-saw term and the necessary correction from the triplet term. To generate such simple structures in m_ν^I, interplay of the parameters in m_D and M_R is required. In a given theory or model this can be natural, but a priori it is more appealing that m_L directly has this simple form.

As already mentioned, we need to fill the zero entries in these matrices via the conventional see-saw term. In what regards the possibility of a discrete LR symmetry, it should be noted that cases (A) and (B) are singular and can not be inverted. Thus, if these matrices correspond to m_L, and if $M_R \propto m_L$, we can not construct the inverse of M_R and the see-saw formula does not apply. Sterile neutrinos are the consequence of such a situation, for recent analyzes see [26]. One will have to omit the simplifying assumption $M_R \propto m_L$ in order to allow for a correction to the leading structure in m_L. On the other hand, the matrix (C) is invertible and can correspond to the triplet term in a discrete LR symmetric theory. Anyway, for simplicity and illustration we will focus here on three rather simple perturbations to the candidate matrices. What we mean by this is that m_ν^I has entries of

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[4] A similar strategy to the one presented here has been discussed in the context of quasi-degenerate neutrinos previously in ref. [10]. It was assumed that m_L is proportional to the unit matrix (made possible by a SO(3) symmetry) and the conventional see-saw term corresponds to sequential dominance [14], thereby generating quasi-degenerate neutrinos with large mixing.

the same order of magnitude, at most differing from each other by order one coefficients. The first possible perturbation is purely anarchical [27]:

$$m_\nu^I \simeq v_L \, \epsilon \begin{pmatrix} a & b & c \\ \cdot & d & e \\ \cdot & \cdot & f \end{pmatrix} . \qquad (4.2)$$

Such a matrix can be obtained if both m_D and M_R are anarchical, or if only one of them is anarchical and the other one proportional to the unit matrix. The second perturbation corresponds to a μ–τ symmetric [25] matrix, i.e., $b = c$ and $d = f$:

$$m_\nu^I \simeq v_L \, \epsilon \begin{pmatrix} a & b & b \\ \cdot & d & e \\ \cdot & \cdot & d \end{pmatrix} . \qquad (4.3)$$

As the candidate matrices in eq. (4.1) are also μ–τ symmetric, adding this perturbation will not change the values $U_{e3} = 0$ and $\theta_{23} = \pi/4$. Such a μ–τ symmetric correction can be achieved when m_D and M_R have a 23 exchange symmetry [28]. The third case occurs when all entries in m_ν^I are identical, i.e., m_ν^I is flavor democratic [29]:

$$m_\nu^I \simeq v_L \, \epsilon \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} . \qquad (4.4)$$

Symmetries such as $S(3)$ can lead to such a structure.

We start with the leading structure (A), corresponding to a normal hierarchy. If it would correspond to the triplet term m_L, one would have $v_L \simeq \sqrt{\Delta m_{\rm A}^2/2}$. The perturbation has to generate the solar mass squared difference, therefore $\epsilon \simeq \sqrt{\Delta m_\odot^2/\Delta m_{\rm A}^2}$. For an anarchical perturbation as in eq. (4.2), one finds naturally large θ_{12}, while $U_{e3} \simeq \epsilon \, (b - c)/\sqrt{8}$, $\theta_{23} - \pi/4 \simeq \epsilon \, (d - f)/4$ and $\Delta m_\odot^2/\Delta m_{\rm A}^2 \propto \epsilon^2$. Hence, both U_{e3} and $\theta_{23} - \pi/4$ are of order $\sqrt{\Delta m_\odot^2/\Delta m_{\rm A}^2}$. If $f = d$ one keeps θ_{23} maximal while $U_{e3} \neq 0$, and for $b = c$ it holds that U_{e3} is zero while $\theta_{23} \neq \pi/4$ [30]. Such a simple possibility does not exist for the other candidates (B) and (C). Both observables remain exactly zero if the type I correction is μ–τ symmetric as in eq. (4.3). Solar neutrino mixing is then naturally of order one: $\sin^2 \theta_{12} \simeq (a - d - e + w)/(2 \, w)$, where $w = \sqrt{8 \, b^2 + (a - d - e)^2}$. Now consider the flavor democratic perturbation from eq. (4.4). One eigenvalues is zero, and one is $3 \, \epsilon \, v_L$ with an eigenvector $(1/\sqrt{3}, \, 1/\sqrt{3}, \, 1/\sqrt{3})^T$ and therefore $\sin^2 \theta_{12} = \frac{1}{3}$. This is of course tri-bimaximal mixing [31]. We will elaborate more on this interesting possibility in section 6.

Let us turn to the inverted hierarchy. If matrix (B) corresponds to m_L, then $v_L \simeq \sqrt{\Delta m_{\rm A}^2/\sqrt{2}}$. The correction to the zeroth order matrix (B) — in absence of charged lepton contributions to the mixing matrix — has to be sizable and tuned. It is however

possible that an anarchical perturbation from m_ν^I, being suppressed with respect to m_L by a small factor of $\epsilon \sim \sqrt{\Delta m_\odot^2 / \Delta m_A^2}$, corrects case (B) in an appropriate way, leading to U_{e3} and $\theta_{23} - \pi/4$ of order $\sqrt{\Delta m_\odot^2 / \Delta m_A^2}$. For a μ–τ symmetric small correction one has that the smallest mass is $(d-e)\,\epsilon$ and of course $U_{e3} = \theta_{23} - \pi/4 = 0$. The ratio of mass squared differences is $\Delta m_\odot^2 / \Delta m_A^2 \simeq \sqrt{2}\,(a+d+e)\epsilon$ and solar neutrino mixing is governed by $\sin\theta_{12} \simeq \sqrt{\frac{1}{2}} - (a-d-e)\,\epsilon/8$. If the order one coefficients conspire such that $(a+d+e) \ll (a-d-e)$, then small $\Delta m_\odot^2$ goes along with non-maximal θ_{12}. This in turn means that a flavor democratic perturbation does not work, since in this case $\Delta m_\odot^2 / \Delta m_A^2 \simeq 3\sqrt{2}\,\epsilon$ and $|U_{e2}| \simeq \sqrt{\frac{1}{2}} - \epsilon/8$. Hence, $\sin\theta_{12} \simeq \sqrt{\frac{1}{2}}\,(1 - \frac{1}{24}\Delta m_\odot^2 / \Delta m_A^2)$, which is too small a value.

Apart from anarchical corrections, note that one needs a type I contribution of the form

$$m_\nu^I = -m_D^T M_R^{-1} m_D \simeq v_L \begin{pmatrix} \mathcal{O}(\lambda) \text{ or } \mathcal{O}(1) & \mathcal{O}(\lambda^{n_1}) & \mathcal{O}(\lambda^{n_2}) \\ \cdot & \mathcal{O}(\lambda^{n_3}) & \mathcal{O}(\lambda^{n_4}) \\ \cdot & \cdot & \mathcal{O}(\lambda^{n_5}) \end{pmatrix}, \qquad (4.5)$$

where n_i is some integer number. This implies non-trivial structures of m_D and/or M_R. For instance, if (other choices are of course possible)

$$m_D = v \begin{pmatrix} a_D\,\lambda^4 & b_D\,\lambda^5 & c_D\,\lambda^5 \\ \cdot & d_D\,\lambda^2 & e_D\,\lambda^2 \\ \cdot & \cdot & f_D\,\lambda \end{pmatrix} \text{ and } M_R = v_R \begin{pmatrix} a_R\,\lambda^7 & 0 & 0 \\ \cdot & d_R\,\lambda^2 & 0 \\ \cdot & \cdot & f_R \end{pmatrix}, \qquad (4.6)$$

we would get

$$m_\nu^I = -m_D^T M_R^{-1} m_D \simeq \frac{v^2}{v_R} \begin{pmatrix} \mathcal{O}(\lambda) & \mathcal{O}(\lambda^2) & \mathcal{O}(\lambda^2) \\ \cdot & \mathcal{O}(\lambda^2) & \mathcal{O}(\lambda^3) \\ \cdot & \cdot & \mathcal{O}(\lambda^2) \end{pmatrix}, \qquad (4.7)$$

which can satisfy the data if added to m_L.

Now, turning to quasi-degenerate neutrinos, assume that matrix (C) corresponds to m_L. An anarchical perturbation allows for successful phenomenology. Diagonalizing matrix (C) plus a flavor democratic perturbation, gives eigenvalues -1, 1 and $1 + 3\epsilon$, where the latter has an eigenvector $(1/\sqrt{3},\, 1/\sqrt{3},\, 1/\sqrt{3})^T$, thereby resembling tri-bimaximal mixing. Recall that to accommodate the data, it is necessary that the neutrino with mass m_2 has this eigenvector. Thus, additional breaking is required (for instance via radiative corrections), in addition also because only one non-zero Δm^2 is present.

Another possibility is the following: since the matrix (C) is invertible, we can assume discrete LR symmetry and thus $m_L \propto M_R$. Choosing for instance

$$m_D = v \begin{pmatrix} a_D\,\lambda^3 & b_D\,\lambda^2 & c_D\,\lambda^2 \\ \cdot & d_D\,\lambda & e_D\,\lambda \\ \cdot & \cdot & f_D \end{pmatrix} \text{ and } M_R = v_R \begin{pmatrix} X & 0 & 0 \\ \cdot & 0 & Y \\ \cdot & \cdot & 0 \end{pmatrix}, \qquad (4.8)$$

gives a low energy mass matrix capable of explaining the data [16, 17]:

$$m_\nu^{II} + m_\nu^I \simeq v_L \begin{pmatrix} X & \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) \\ \cdot & \mathcal{O}(\lambda^2) & Y \\ \cdot & \cdot & \mathcal{O}(\lambda) \end{pmatrix}, \tag{4.9}$$

where only the leading terms are given. The Dirac mass matrix resembles the up-quarks and the form of M_R is trivial to obtain if the heavy neutrinos N_1, N_2 and N_3 have the charges 0, 1 and -1 under $L_\mu - L_\tau$ [16].

We were using in this section, in particular for the normal hierarchy, mainly a more or less anarchical perturbation generated by the type I see-saw term, which is somewhat incompatible with the naive expectation of hierarchical Dirac mass matrices and also with a discrete LR symmetry. In the next section we will show that it is also possible to perturb a given mixing scenario when both hierarchical Dirac mass matrices and discrete LR symmetry are present. In this case the zeroth order mass matrix as provided by m_L has to have a more complicated form.

5. Deviations from bimaximal and tri-bimaximal mixing

In the last section we have perturbed very simple mass matrices leading to $U_{e3} = 0$ and $\theta_{23} = \pi/4$ via more or less anarchical perturbations from m_ν^I. In particular, m_D was required to possess a rather unusual structure. We show in this section an alternative possibility to deviate (in the normal hierarchy) within the type II see-saw mechanism certain neutrino mixing scenarios, such as bimaximal [32] or tri-bimaximal [31] mixing. The difference with respect to the proposals in section 4 is that the zeroth order mass matrix, as provided by m_L has a more complicated structure. One can again imagine that these simple scenarios are implemented by some symmetry only in m_L, whereas the other mass matrices are connected to the "hierarchical with small mixing" form known from the quarks. In contrast to section 4, the perturbation generated by the conventional see-saw term works with a discrete LR symmetry and also with a hierarchical Dirac mass matrix [9]. Both the bimaximal and tri-bimaximal scenario predict vanishing θ_{13} and $\cos 2\theta_{23}$, therefore they are special cases of μ–τ symmetry [25, 28, 16]. In general, the procedure described here will be possible for any μ–τ symmetric mixing scenario, but for definiteness we stick to bimaximal and tri-bimaximal mixing. The latter is in perfect agreement with current data and a perturbation due to type II see-saw (or some other mechanism) is strictly speaking not necessary, but will lead to non-vanishing θ_{13} and $\cos 2\theta_{23}$. In contrast to this, bimaximal mixing is ruled out by several standard deviations, and therefore requires a perturbation. As we will show, this perturbation can mimic Quark-Lepton Complementarity.

Let us start with bimaximal mixing [32], defined as

$$U_{\text{bimax}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{5.1}$$

corresponding to $\theta_{12} = \theta_{23} = \pi/4$, $U_{e3} = 0$ and leading to a mass matrix

$$m_\nu^{\text{bimax}} = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+D) & \frac{1}{2}(A-D) \\ \cdot & \cdot & \frac{1}{2}(A+D) \end{pmatrix}, \qquad (5.2)$$

where

$$A = \frac{m_1^0 + m_2^0}{2}, \; B = \frac{m_2^0 - m_1^0}{2\sqrt{2}}, \; D = m_3^0 \,. \qquad (5.3)$$

The superscript 0 indicates that these are the initial mass eigenvalues, valid before a perturbation from the type II see-saw term is switched on. We demonstrate now how the type II see-saw mechanism can lead to a deviation from bimaximal mixing in accordance with neutrino data. We can assume again discrete LR symmetry, eq. (2.7), where $v_L f$ is now given by eq. (5.2). The inverse of M_R is given by

$$M_R^{-1} = \frac{v_L}{v_R} \, m_L^{-1} = \frac{v_L}{v_R} \begin{pmatrix} \tilde{A} & \tilde{B} & \tilde{B} \\ \cdot & \frac{1}{2}(\tilde{A}+\tilde{D}) & \frac{1}{2}(\tilde{A}-\tilde{D}) \\ \cdot & \cdot & \frac{1}{2}(\tilde{A}+\tilde{D}) \end{pmatrix}, \qquad (5.4)$$

where

$$\tilde{A} = \frac{A}{A^2 - 2B^2}, \; \tilde{B} = \frac{-B}{A^2 - 2B^2}, \; \tilde{D} = \frac{1}{D} \,. \qquad (5.5)$$

We shall assume in the following a normal hierarchical mass spectrum, i.e., $(m_3^0)^2 \gg (m_{1,2}^0)^2$. For zero m_1^0 the mass matrix eq. (5.2) would be singular. Assuming that m_D is hierarchical can be quantified as $m_D \simeq \text{diag}(0,0,m)$. It is then easy to show that the effect of the conventional see-saw term is only [5, 9]:

$$m_D^T M_R^{-1} m_D \simeq \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 0 & 0 \\ \cdot & 0 & s \end{pmatrix}, \; \text{where } s \equiv v_L^2 \, \frac{m^2}{4\gamma\, v^2} \left(\frac{1}{m_1^0} + \frac{1}{m_2^0} + \frac{2}{m_3^0} \right) \,. \qquad (5.6)$$

This term has to be subtracted from m_L which is given in eq. (5.2). The zero entries in this matrix can also be small and suppressed with respect to the 33 element without changing our conclusions. With $\gamma \simeq 1$, $m \simeq v$ and one of the m_i^0 of order v_L, this conventional term is of similar magnitude as the triplet contribution, which is proportional to v_L. Hence, identifying m_D with the charged leptons or the down-quarks will lead to a negligible correction of m_ν^I to m_ν^{II} if $\gamma = \mathcal{O}(1)$. If however m_D is related to the up-quarks, $m \simeq v$, then we can estimate this term as

$$s \simeq \frac{0.1}{4\,\gamma} \left(\frac{v_L}{10^{-2}\;\text{eV}} \right)^2 \left(\frac{10^{-3}\;\text{eV}}{m_1^0} \right) \; \text{eV} \,, \qquad (5.7)$$

where again hierarchical m_i^0 were assumed. Leaving LR symmetry aside, many non-singular mass matrices M_R in connection with hierarchical Dirac mass matrices will have the 33

entry of m_ν^I as the leading term and can be cast in the form (5.6). Naturally, for reference values $m_1^0 = 10^{-3}\,\text{eV}$ and $v_L = 10^{-2}\,\text{eV}$, the order of s can be — without varying γ around the value one within more than one order of magnitude — given by the scale of neutrino masses $\sqrt{\Delta m_\odot^2}$ or $\sqrt{\Delta m_A^2}$.

We can now diagonalize the perturbed mass matrix. For s of order D or smaller and for $D^2 \gg A^2, B^2$ the mixing angles are given by

$$|U_{e3}| \simeq \frac{B\,s}{\sqrt{2}\,D^2}\,, \quad \sin^2\theta_{23} \simeq \frac{1}{2}\left(1 + \frac{s}{D}\right)\,, \quad \tan 2\theta_{12} \simeq 4\sqrt{2}\,\frac{B}{s}\,. \tag{5.8}$$

From the expression for θ_{12} and assuming hierarchical m_i^0, one obtains that $|s| \sim |m_2^0| \sim \sqrt{\Delta m_\odot^2}$ in order to reproduce the observations. One interesting aspect, which we will assume now, is the following: from eqs. (5.2) and (5.3) it is obvious that for $m_1^0 = -m_2^0$, or $A = 0$, one would start with vanishing $\Delta m_\odot^2$. In this case the conventional see-saw term s creates not only the required deviation from maximal solar neutrino mixing, but induces also the solar mass squared difference, which is then proportional to s^2. The phenomenological relation that the deviation from maximal solar neutrino mixing is of the same order as $\sqrt{\Delta m_\odot^2/\Delta m_A^2}$ can thereby be explained, since the same parameter is responsible for both deviations.

We can discuss also a possible connection to Quark-Lepton Complementarity (QLC) [33]. The deviation from maximal solar neutrino mixing can empirically be written as [34] $U_{e2} = \sqrt{1/2}\,(1 - \lambda)$, where $\lambda \simeq 0.22$ quantifies the required deviation. If not a coincidence, the parameter λ is the sine of the Cabibbo angle θ_C and therefore [33] $\theta_{12} + \theta_C = \pi/4$. In this case $\tan 2\theta_{12} = 1/(2\lambda) + \mathcal{O}(\lambda)$, and from comparing eq. (5.8) with this expression it follows that QLC is mimicked[5] when $s/B \simeq 8\sqrt{2}\,\lambda$. In order to distinguish the type I contribution to bimaximal mixing from QLC, we note that there are two main scenarios in which QLC can arise [33] (a recent detailed analysis of the low and high energy phenomenology of these two scenarios has been conducted in [35]). Their most important and most easily testable difference is that one scenario predicts $|U_{e3}|^2 = \lambda^2/2 \simeq 0.03$, while the other one predicts $|U_{e3}| = A\,\lambda^2/\sqrt{2} \simeq 0.03$, where A is a parameter in the Wolfenstein parametrization of the CKM matrix. In our framework, one finds that $|U_{e3}|$ is of similar size than in the second QLC scenario, but obeys the correlation

$$\frac{2\,|U_{e3}|}{\tan 2\theta_{12}} \simeq \left(\sin^2\theta_{23} - \frac{1}{2}\right)^2 \simeq \frac{\Delta m_\odot^2}{\Delta m_A^2}\cos 2\theta_{12}\,. \tag{5.9}$$

Since this relation is not predicted by the QLC scenario, we can in principle distinguish it from our scenario.

Neutrino mixing can also be very well described by tri-bimaximal mixing [31], which is defined by the mixing matrix in eq. (6.1). The resulting mass matrix $m_\nu = U^*\,m_\nu^{\text{diag}}\,U^\dagger$

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[5]Strictly speaking, every model predicting $\sin^2\theta_{12} \simeq 0.28$ mimics QLC, in the sense that this is about the prediction of $\theta_{12} = \pi/4 - \theta_C$. What we mean here by mimicking QLC is that one gets from bimaximal mixing to $\sin^2\theta_{12} \simeq 0.3$.

is

$$m_\nu = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}, \tag{5.10}$$

where

$$A = \frac{1}{3}(2\,m_1^0 + m_2^0)\,, \quad B = \frac{1}{3}(m_2^0 - m_1^0)\,, \quad D = m_3^0\,, \tag{5.11}$$

or $m_1^0 = A - B$ and $m_2^0 = A + 2\,B$. For normal hierarchical neutrinos we have $D^2 \gg A^2, B^2$. Note that if we remove B from the 23 block of m_ν we obtain eq. (5.2), i.e., bimaximal mixing. Suppose again that the mass matrix (5.10) corresponds to m_ν^{II}. In analogy to the example for bimaximal mixing given above the conventional see-saw term will result in a small contribution to the 33 entry. The results for U_{e3} and $\theta_{23} - \pi/4$ are similar to the case of initial bimaximal mixing discussed above, while for solar neutrino mixing it holds

$$\tan 2\theta_{12} \simeq \frac{2\sqrt{2}}{1 - s/(2B)}\,. \tag{5.12}$$

A slightly smaller s is required in this case, which can be expected, since tri-bimaximal mixing is very close to current data and little room for deviations is there.

6. More on tri-bimaximal mixing

We have seen in section 4 that a sum of two relatively simple matrices can lead to (close to) tri-bimaximal mixing in the normal hierarchy. We will now comment more on the realizations of this mixing scheme within the type II see-saw, discussing also the inverted hierarchy and quasi-degenerate neutrinos. Tri-bimaximal mixing is defined as [31]

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}\,. \tag{6.1}$$

It corresponds to $\sin^2\theta_{12} = 1/3$, $U_{e3} = 0$ and $\theta_{23} = \pi/4$. The resulting mass matrix $m_\nu = U^* m_\nu^{\text{diag}} U^\dagger$ can be written in terms of matrices multiplied with the masses:

$$m_\nu = \frac{m_1}{6} \begin{pmatrix} 4 & -2 & -2 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + \frac{m_2}{3} \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + \frac{m_3}{2} \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{pmatrix}\,. \tag{6.2}$$

This equation is exact, i.e., there are no order one coefficients involved. We remark here that such a sum of three matrices could also be realized if there are three different contributions to the effective mass matrix, such as the ones mentioned at the end of section 2.2. When we assume a normal hierarchy and neglect m_1 it follows

$$m_\nu = \frac{\sqrt{\Delta m_\odot^2}}{3} \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + \frac{\sqrt{\Delta m_A^2}}{2} \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{pmatrix}\,, \tag{6.3}$$

i.e., the second term dominates. One of the terms could stem from the triplet term and the other one from the conventional see-saw term. In section 4 we encountered this matrix when we perturbed a triplet term corresponding to the atmospheric neutrino mass scale (matrix (A) in eq. (4.1)) with a flavor democratic type I term. Another possibility is that the triplet term is subleading and corresponds to the flavor democratic contribution proportional to $\sqrt{\Delta m_\odot^2}$. The second, leading term is then generated by the conventional see-saw mechanism, for instance via sequential dominance or conservation of L_e. Note however that a democratic mass matrix has rank 1 and can not be inverted. Hence, if there is a discrete LR symmetry then the term proportional to the democratic matrix can not stem from the triplet but must come from the conventional see-saw term. Further note that since the non-vanishing entries of the second term are identical, there will be additional symmetries, such as S_2 or Z_2, required. For instance, if the democratic term is generated by a triplet term, then the second term could stem from $M_R \propto \mathrm{diag}(0,0,1)$ and for m_D it suffices that the third row looks like $(0,-1,1)$.

Another possibility is that there are similar contributions of the type I and triplet term. We can then discuss the inverted hierarchy and quasi-degenerate neutrinos. With $m_3 = 0$ and equal CP parities of the remaining states, i.e., $m_1 = m_2$, we can write

$$\frac{m_\nu}{\sqrt{\Delta m_{\mathrm{A}}^2}} \simeq \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 1 & 0 \\ \cdot & \cdot & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 0 & 1 \\ \cdot & \cdot & 0 \end{pmatrix} . \tag{6.4}$$

Note that m_ν^I and m_ν^{II} have to have almost the same size. Strictly speaking, $\theta_{12} = 0$ results from this matrix. The underlying reason is the simplifying assumption $m_1 = m_2$, for which small corrections (of order $\Delta m_\odot^2/m_1^2$) are neglected. However, the quasi-degeneracy of the two neutrino masses can easily lead to large solar neutrino mixing once small breaking parameters are introduced. Breaking is necessary anyway in order to generate the solar mass splitting. Eq. (6.4) is a sum of a unit matrix and a matrix conserving $L_\mu - L_\tau$ (plus an additional symmetry making the 11 and 23 entries identical). The first matrix could be a triplet term, generated with SO(3), and the second term could stem from a type I see-saw with a diagonal m_D and a M_R of the form

$$M_R = M \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 0 & 1 \\ \cdot & \cdot & 0 \end{pmatrix} .$$

We encountered this kind of contribution from m_ν^I already at the end of section 4, see the remarks after eq. (4.9). It could also be that there is a discrete LR symmetry, in which m_L and M_R are proportional to the unit matrix. A very unusual form of m_D is then required in order to obtain the second term in eq. (6.4). We could also generate this scenario when both m_D and M_R are proportional to the unit matrix and the triplet term obeys $L_\mu - L_\tau$. For an inverted hierarchy with opposite CP parities, we have

$$\frac{m_\nu}{\sqrt{\Delta m_{\mathrm{A}}^2}} \simeq -\frac{2}{3} \begin{pmatrix} 0 & 1 & 1 \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ \cdot & -\frac{1}{2} & -\frac{1}{2} \\ \cdot & \cdot & -\frac{1}{2} \end{pmatrix} , \tag{6.5}$$

where both terms correspond to $L_e - L_\mu - L_\tau$. This matrix corresponds to tri-bimaximal mixing and requires small breaking in order to generate the solar mass splitting.

Similar discussions are possible for quasi-degenerate neutrinos. If $m_1 = m_3 = -m_2$, then we can write

$$\frac{m_\nu}{m_1} \simeq \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 1 & 0 \\ \cdot & \cdot & 1 \end{pmatrix} - \frac{2}{3} \begin{pmatrix} 0 & 1 & 1 \\ \cdot & 0 & 1 \\ \cdot & \cdot & 0 \end{pmatrix} \text{ or } \frac{m_\nu}{m_1} \simeq \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 1 \\ \cdot & 0 & 1 \\ \cdot & \cdot & 0 \end{pmatrix} . \quad (6.6)$$

We can therefore express the mass matrix as a sum of a triangular matrix and a unit (or a democratic) matrix. For CP parities leading to $m_1 = -m_3 = -m_2$ we can decompose the mass matrix as

$$\frac{m_\nu}{m_1} \simeq -\frac{2}{3} \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 0 & 1 \\ \cdot & \cdot & 0 \end{pmatrix} , \quad (6.7)$$

where flavor democracy and $L_\mu - L_\tau$ seem to play a role again. Discrete LR symmetry is again possible. The last two cases do not produce tri-bimaximal mixing, but can easily do so for appropriate small breaking parameters of order $\Delta m_\odot^2/m_1$ and $\Delta m_A^2/m_1$.

7. Summary

Both the type I and the type II see-saw mechanism explain tiny neutrino masses, but large neutrino mixing is not predicted per se, unless there is additional input. While generating large neutrino mixing is well and often studied within the conventional (type I) see-saw mechanism, large or maximal mixing within the type II see-saw received so far little attention. Therefore we discussed in this article the interplay of both terms of the type II see-saw in order to understand the unexpected features of neutrino mixing. The fact that the neutrino mass matrix is in this case a sum of two terms opens up the possibility of cancellation if the two terms are comparable. It is also possible that the sum of two terms generates the unexpected features of neutrino mixing. Alternatively and most natural, one term can be the leading contribution, while the other one can give perturbations. In this context, several possibilities were suggested in this article:

- we introduced "type II enhancement", i.e., showed that within type II see-saw models mild cancellation of certain terms can lead to the generation of large mixing angles, even though all individual matrices involved predict small mixing. Both discrete and gauge LR symmetry are possible. A hint to obtain such models is to note that the complete 6×6 neutrino mass matrix can have a vanishing determinant. The requirement that there is similar, but small, mixing in both m_D and M_R differs from the usual (type I) see-saw enhancement of neutrino mixing, which requires a stronger hierarchy in the heavy neutrino sector and somewhat decouples the two sectors. Maximal or vanishing mixing requires additional input, such as the equality of certain mass matrix elements;

- the leading structure of the neutrino mass matrix as displayed in eq. (4.1) can be generated by some symmetry acting on m_L. Necessary corrections stem from the conventional see-saw term. However, in case of a normal and inverted hierarchy the leading structures given in eq. (4.1) (corresponding to L_e and $L_e - L_\mu - L_\tau$, respectively) are singular, which make these scenarios incompatible with the discrete LR symmetric relation $m_L \propto M_R$. In contrast to this, the leading structure for quasi-degenerate neutrinos can be generated by the unit matrix or via the matrix in eq. (2.3), corresponding to $L_\mu - L_\tau$. They can be inverted and are compatible with discrete LR symmetry. We showed that anarchical perturbations can generate successful phenomenology from the zeroth order matrices and that a μ–τ symmetric perturbation keeps the initial values of zero U_{e3} and maximal θ_{23};

- one could imagine that the triplet term has a more complicated structure corresponding to bimaximal or tri-bimaximal mixing. In discrete and gauge LR symmetric scenarios with hierarchical Dirac mass matrices it is easily possible that a small perturbation to m_L arises, which deviates the mixing scenarios. Quark-Lepton Complementarity could be mimicked, and in addition the empirical relation $1 - \sqrt{2} \sin\theta_{12} \simeq \sqrt{\Delta m_\odot^2 / \Delta m_A^2}$ can be explained if m_L alone would generate vanishing $\Delta m_\odot^2$;

- we realized that for tri-bimaximal mixing the light neutrino mass matrix can often be written as a sum of two terms both of which have an interesting structure. We interpret this by assuming that each term stems from one of the two terms in the type II see-saw formula. For a normal hierarchy, the two contributions have different order of magnitude, their ratio is given by $\sqrt{\Delta m_\odot^2 / \Delta m_A^2}$. For the inverted hierarchy and for quasi-degenerate neutrinos, they have to have similar size.

The next generation of experiments will show if θ_{13} is small or tiny and if θ_{23} is large or close to maximal. The discussed (incomplete) list of scenarios shows how the interference of the two terms in the type II see-saw leads to various interesting possibilities to understand all possibilities.

Acknowledgments

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